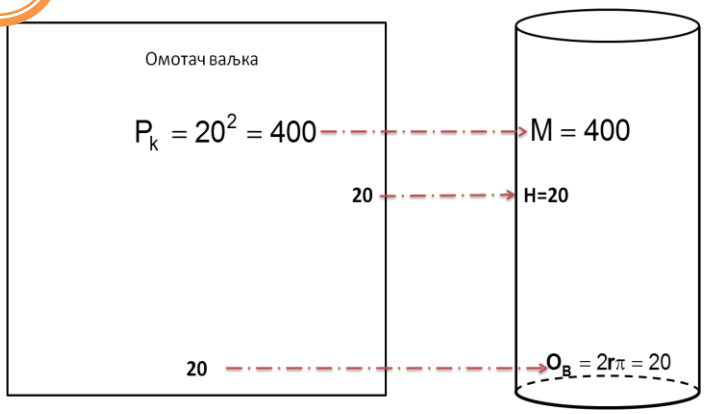
1.

$P = 28\pi$
$H : r \Rightarrow 5 : 2$
$V = ?$
$H = 5k$
$r = 2k$
$P = 2r\pi(r + H)$
$28\pi = 4k\pi(7k)$
$28 = 28k^2$
$k^2 = 1 \Rightarrow k = 1$

$H = 5k = 5$
$r = 2k = 2$
$B = r^2\pi = 4\pi$
$V = BH = 4\pi 5 = 20\pi$

2.



$2r\pi = 20 \Rightarrow r = \frac{20}{2\pi} = \frac{10}{\pi}$

$B = r^2\pi = \frac{100}{\pi^2}\pi = \frac{100}{\pi} \approx 31{,}85cm^2$

За дно је потребно парче лима површине $31{,}85см^2$.

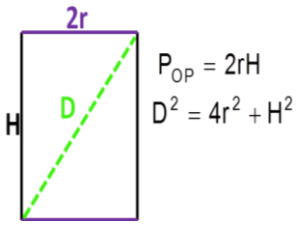
$V = BH$

$V \approx 31{,}85 \cdot 20 \approx 637cm^3 = 0{,}637dm^3 = 0{,}637L$

Запремина лонца је 0,637 литара.

3.

$P_{OP} = 120cm^2$
$D = 17cm$
$P, V = ?$



$P_{OP} = 2rH$
$120 = 2rH$
$rH = 60$

$D^2 = 4r^2 + H^2$
$289 = 4r^2 + H^2$

$rH = 60$
$4r^2 + H^2 = 289$
$4r^2 + 4rH + H^2 = 289 + 4 \cdot 60$
$(2r + H)^2 = 529 = 23^2$
$\Rightarrow 2r + H = 23 \Rightarrow H = 23 - 2r$
$rH = 60$
$r(23 - 2r) = 60$
$-2r^2 + 23r - 60 = 0$
$r_{1/2} = \frac{-23 \pm \sqrt{529 - 480}}{-4}$
$r_{1/2} = \frac{-23 \pm 7}{-4}$
$r_1 = \frac{15}{2}; r_2 = 4$

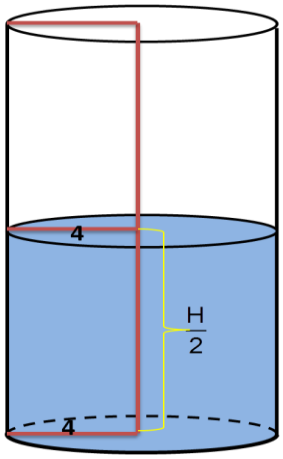
$(1) r = \frac{15}{2}$
$H = 23 - 2r = 8$
$B = r^2\pi = \frac{225}{4}\pi cm^2$
$M = 2rH\pi = 120\pi cm^2$
$P = 2B + M = \frac{465}{2}\pi cm^2$
$V = BH = 450\pi cm^3$

$(2) r = 4$
$H = 23 - 2r = 15$
$B = r^2\pi = 16\pi cm^2$
$M = 2rH\pi = 120\pi cm^2$
$P = 2B + M = 152\pi cm^2$
$V = BH = 240\pi cm^3$

4.

$2r = 8 \Rightarrow r = 4cm$

$a_T = 6cm$



$$V_{vode} = r^2\pi \cdot \frac{H}{2}$$

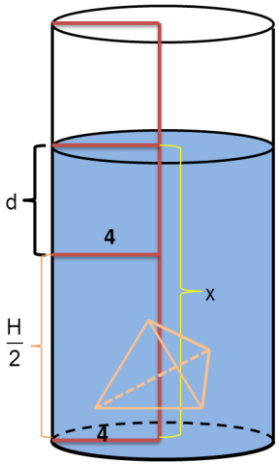
$$V_{vode} = 16\pi \cdot \frac{H}{2} = 8\pi H$$

тетраедар

(пирамида-задатак 8)

$$V_T = \frac{a_T^3\sqrt{2}}{12}$$

$$V_T = \frac{216\sqrt{2}}{12} = 18\sqrt{2}cm^3$$



$d = ?$

$$V = V_{vode} + V_T = r^2\pi x$$

$$8\pi H + 18\sqrt{2} = 16\pi x$$

$$x = \frac{8\pi H + 18\sqrt{2}}{16\pi}$$

$$x = \frac{8\pi H}{16\pi} + \frac{18\sqrt{2}}{16\pi}$$

$$x = \frac{H}{2} + \frac{9\sqrt{2}}{8\pi}$$

$$d = x - \frac{H}{2} = \frac{9\sqrt{2}}{8\pi}cm$$

5.

$r_1 = H_2$

$r_2 = H_1$

$P_1 + P_2 = 50\pi$

$V_1 + V_2 = 30\pi$

$P_1, P_2, V_1, V_2 = ?$

$$P_1 = 2r_1^2\pi + 2r_1H_1\pi$$
$$P_1 = 2r_1^2\pi + 2r_1r_2\pi$$
$$P_2 = 2r_2^2\pi + 2r_2H_2\pi$$
$$P_2 = 2r_2^2\pi + 2r_2r_1\pi$$
$$P_1 + P_2 = 50\pi$$
$$2r_1^2\pi + 2r_1r_2\pi + 2r_2^2\pi + 2r_2r_1\pi = 50\pi \quad /:2\pi$$
$$r_1^2 + r_1r_2 + r_2^2 + r_1r_2 = 25$$
$$r_1^2 + 2r_1r_2 + r_2^2 = 25$$
$$(r_1 + r_2)^2 = 25$$
$$r_1 + r_2 = 5$$

$$V_1 = r_1^2H_1\pi$$
$$V_1 = r_1^2r_2\pi$$
$$V_2 = r_2^2H_2\pi$$
$$V_2 = r_2^2r_1\pi$$
$$V_1 + V_2 = 30\pi$$
$$r_1^2r_2\pi + r_2^2r_1\pi = 30\pi \quad /:\pi$$
$$r_1^2r_2 + r_2^2r_1 = 30$$
$$r_1r_2(r_1 + r_2) = 30$$
$$r_1r_2 \cdot 5 = 30$$
$$r_1r_2 = 6$$

$r_1 + r_2 = 5$

$r_1 + r_2 = 6$

$(1)\ r_1 = 2 = H_2$

$r_2 = 3 = H_1$

$(2)\ r_1 = 3 = H_2$

$r_2 = 2 = H_1$

$(1)\ r_1 = 2; H_1 = 3$

$P_1 = 2r_1\pi(r_1 + H_1) = 20\pi$

$P_2 = 50\pi - 20\pi = 30\pi$

$V_1 = r_1^2H_1\pi = 12\pi$

$V_2 = 30\pi - 12\pi = 18\pi$

$(2)\ r_1 = 3; H_1 = 2$

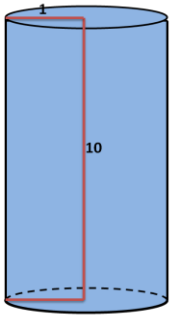
$P_1 = 2r_1\pi(r_1 + H_1) = 30\pi$

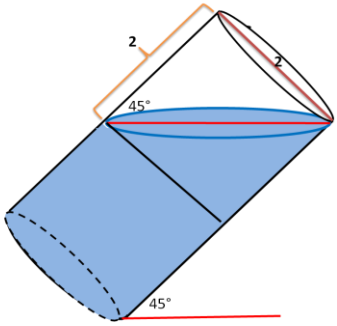
$P_2 = 50\pi - 30\pi = 20\pi$

$V_1 = r_1^2H_1\pi = 18\pi$

$V_2 = 30\pi - 18\pi = 12\pi$

6.





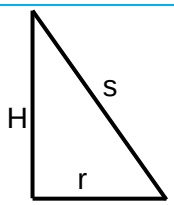
$$V_{prosute\_vode} = \frac{1}{2}V_{valjka(r=1,H=2)} = V$$

$$V_{prosute\_vode} = \frac{1}{2}1^2 \cdot 2\pi = \pi cm^3$$

7.

$P = 96\pi cm^2$
$s = 10cm$
$V = ?$

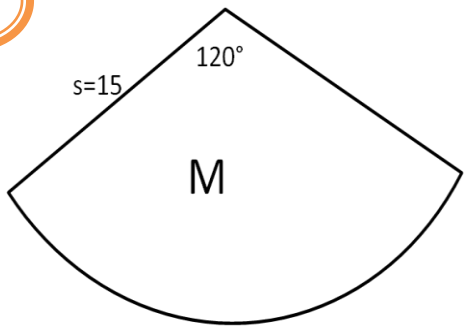
$P = r\pi(r+s)$
$96\pi = r\pi(r+10)$
$r^2 + 10r - 96 = 0$
$r_1 = -16(\perp)$
$r_2 = 6 \Rightarrow r = 6$



$s^2 = r^2 + H^2$
$100 = 36 + H^2$
$H^2 = 64 \Rightarrow H = 8$

$V = \frac{1}{3}r^2H\pi$
$V = \frac{1}{3}36 \cdot 8\pi = 96\pi cm^3$

8.

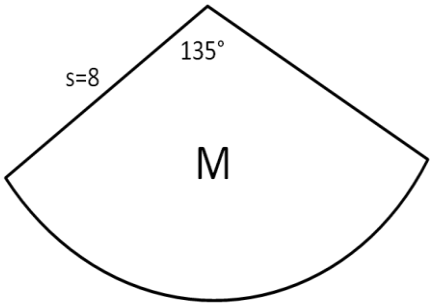


$M = \frac{120°}{360°}s^2\pi = \frac{1}{3}225\pi = 75\pi$
$M = sr\pi$
$75\pi = 15r\pi$
$r = \frac{75}{15} = 5$
$H^2 = s^2 - r^2 = 225 - 25 = 200$
$H = \sqrt{200} = 10\sqrt{2}$

$B = r^2\pi = 25\pi$
$P = B + M = 25\pi + 75\pi$
$P = 100\pi cm^2$

$V = \frac{1}{3}BH = \frac{1}{3}25\pi 10\sqrt{2}cm^3$
$V = \frac{250}{3}\pi\sqrt{2}$

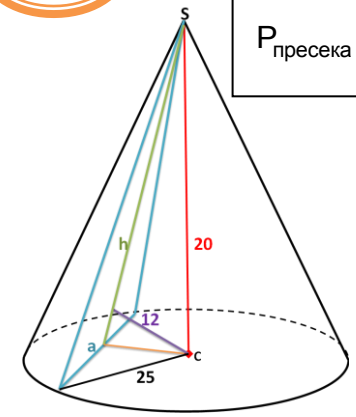
9.



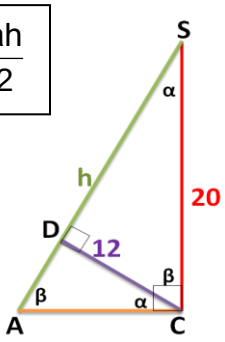
$M = \frac{135°}{360°}s^2\pi = \frac{3}{8}64\pi = 24\pi$
$M = sr\pi$
$24\pi = 8r\pi$
$r = \frac{24}{8} = 3$

Потребан је круг полупречника 3см.

10.



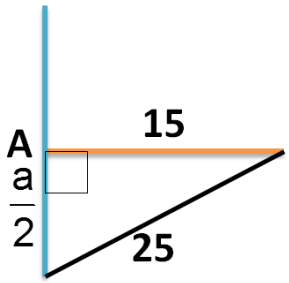
$P_{пресека} = \frac{ah}{2}$



$|SD|^2 = 20^2 - 12^2 = 256$
$|SD| = 16$

Троуглови ACD и CDScy слични
($\sphericalangle ACD \cong \sphericalangle CSD$- углови са нормалним крацима)
$|CD| : |SD| = |AD| : |CD|$
$12 : 16 = |AD| : 12$
$|AD| = \frac{12 \cdot 12}{16} = 9$
$h = |AD| + |SD| = 16 + 9 = 25$

$|AC|^2 = h^2 - |CS|^2$
$|AC|^2 = 625 - 400 = 225$



$$\left(\frac{a}{2}\right)^2 = 25^2 - AC^2 = 625 - 225 = 400$$

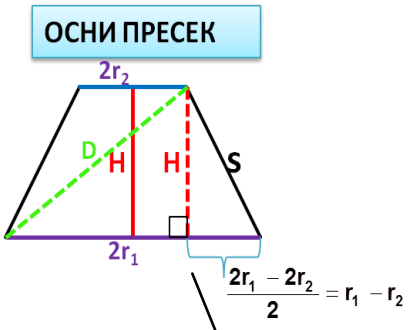
$$\left(\frac{a}{2}\right)^2 = 400 \Rightarrow \frac{a}{2} = 20$$

$$a = 40$$

$$P_{пресека} = \frac{4025}{2} = 500cm^2$$

11.

$P_{op} = 110cm^2$
$H = 1{,}5cm$
$s = 2{,}5cm$
$P, V = ?$



$$P_{op} = \frac{2r_1 + 2r_2}{2} H = (r_1 + r_2) H$$

$$110 = (r_1 + r_2) \cdot 1{,}5$$

$$r_1 + r_2 = \frac{110}{1{,}5} = \frac{220}{3}$$

$$s^2 = (r_1 - r_2)^2 + H^2$$

$$6{,}25 = (r_1 - r_2)^2 + 2{,}25$$

$$(r_1 - r_2)^2 = 4$$

$$r_1 - r_2 = 2$$

$$+ \begin{cases} r_1 + r_2 = \frac{220}{3} \\ r_1 - r_2 = 2 \end{cases}$$

$$2r_1 = \frac{226}{3} \Rightarrow r_1 = \frac{113}{3}$$

$$r_2 = \frac{220}{3} - \frac{113}{3} = \frac{107}{3}$$

$$B_1 = r_1^2 \pi = \frac{226^2}{9} \pi \approx 5675{,}11\pi cm^2$$

$$B_2 = r_2^2 \pi = \frac{107^2}{9} \pi \approx 1272{,}11\pi cm^2$$

$$M = (r_1 + r_2) s\pi = \frac{327}{3} 2{,}5\pi = 272{,}5\pi cm^2$$

$$P = B_1 + B_2 + M \approx 7219{,}72\pi cm^2$$

$$V = \frac{H\pi}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

$$V = \frac{1{,}5\pi}{3}\left(\frac{113^2}{9} + \frac{113107}{9} + \frac{107^2}{9}\right)$$

$$V \approx 2017{,}17\pi cm^3$$

12.

$s = 5cm$
$r_1 - r_2 = 3cm$
$M = \frac{1}{2}P \Rightarrow P = 2M$
$V = ?$

$$s^2 = H^2 + (r_1 - r_2)^2$$

$$25 = H^2 + 9 \Rightarrow H^2 = 16$$

$$H = 4$$

$$P = B_1 + B_2 + M$$

$$2M = B_1 + B_2 + M$$

$$M = B_1 + B_2$$

$$(r_1 + r_2) s\pi = r_1^2 \pi + r_2^2 \pi$$

$$5(r_1 + r_2) = r_1^2 + r_2^2$$

$$r_1 - r_2 = 3 \Rightarrow r_1 = r_2 + 3$$

$$5(r_2 + 3 + r_2) = (r_2 + 3)^2 + r_2^2$$

$$10r_2 + 15 = r_2^2 + 6r_2 + 9 + r_2^2$$

$$2r_2^2 - 4r_2 - 6 = 0$$

$$r_2^2 - 2r_2 - 3 = 0$$

$$r_2 = -1(\perp)$$

$$r_2 = 3$$

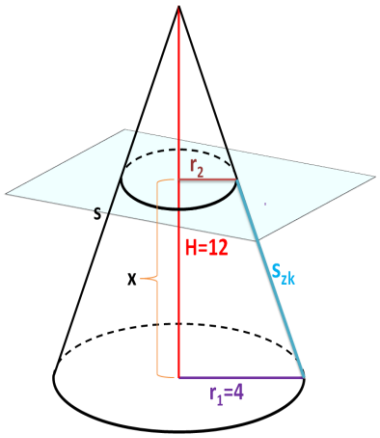
$$r_1 = r_2 + 3 = 6$$

$$V = \frac{H\pi}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

$$V = \frac{4\pi}{3}(36 + 18 + 9)$$

$$V = 84\pi cm^3$$

13.



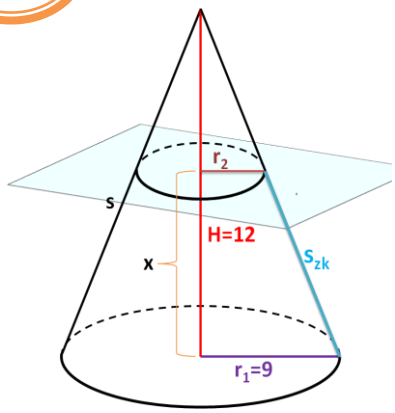
$s^2 = H^2 + r_1^2$
$s^2 = 144 + 16 = 160$
$s = 4\sqrt{10}$

из сличности троуглова:
$r_1 : r_2 = s : (s - s_{zk})$
$4 : r_2 = 4\sqrt{10} : (4\sqrt{10} - s_{zk})$
$4(4\sqrt{10} - s_{zk}) = 4\sqrt{10}r_2$
$16\sqrt{10} - 4s_{zk} = 4\sqrt{10}r_2$
$s_{zk} = \dfrac{16\sqrt{10} - 4\sqrt{10}r_2}{4}$
$s_{zk} = 4\sqrt{10} - \sqrt{10}r_2$
$s_{zk} = \sqrt{10}(4 - r_2)$

$M_{zk} = (r_1 + r_2)s_{zk}\pi$
$15\pi\sqrt{10} = (4 + r_2)\sqrt{10}(4 - r_2)\pi$
$15 = 16 - r_2^2 \Rightarrow r_2^2 = 1$
$r_2 = 1$

из сличности троуглова:
$r_1 : r_2 = H : (H - x)$
$4 : 1 = 12 : (12 - x)$
$4(12 - x) = 12$
$48 - 4x = 12$
$4x = 36$
$x = 9cm$
Раван је од основе удаљена 9см.

14.



(3) из сличности $\triangle$:
$r_1 : r_2 = s : s_2$
$9 : r_2 = 15 : s_2$
$9s_2 = 15r_2 / : 3$
$3s_2 = 5r_2$
$s_2 = \dfrac{5r_2}{3}$

(4) $M_{целе\_купе} : M_{горње\_купе} = 9 : 1$
$r_1 s\pi : r_2 s_2 \pi = 9 : 1$
$9 \cdot 15 : r_2 s_2 = 9 : 1$
$9r_2 s_2 = 9 \cdot 15$
$r_2 s_2 = 15$

(5) $s_2 = \dfrac{5r_2}{3}$
$r_2 s_2 = 15$
$\dfrac{5r_2^2}{3} = 15$
$r_2^2 = 9 \Rightarrow r_2 = 3$

(6) из сличности троуглова:
$r_1 : r_2 = H : (H - x)$
$9 : 3 = 12 : (12 - x)$
$9(12 - x) = 36$
$12 - x = 4$
$x = 8$
Висина зарубљене купе је 8см.

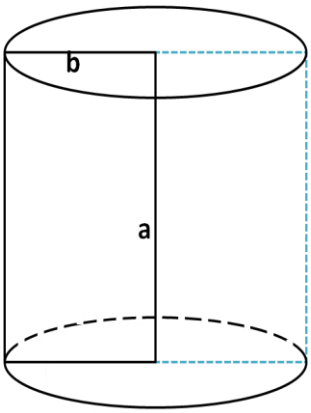
(1) $M_{зарубљене\_купе} : M_{горње\_купе} = 8 : 1$
$M_{целе\_купе} = M_{зарубљене\_купе} + M_{горње\_купе}$
$M_{целе\_купе} : M_{горње\_купе} = 9 : 1$

(2) $s^2 = H^2 + r_1^2$
$s^2 = 144 + 81 = 225$
$s = 15$

$V_{zk} = \dfrac{H\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)$
$V_{zk} = \dfrac{8\pi}{3}(81 + 27 + 9)$
$V_{zk} = 312\pi cm^3$

15.

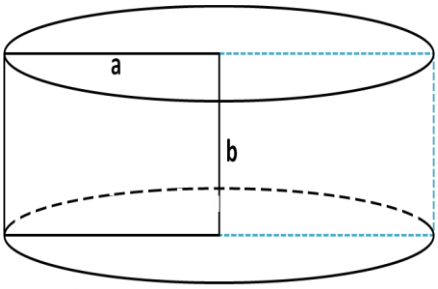
Око странице a:



Ваљак:

$r = b$
$H = a$
$P_1 = 2r\pi(r + H)$
$P_1 = 2b\pi(a + b)$
$V_1 = BH = r^2H\pi$
$V_1 = ab^2\pi$

Око странице b:



Ваљак:

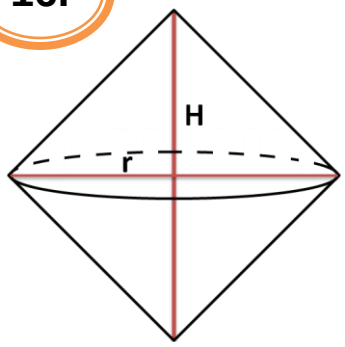
$r = a$
$H = b$
$P_2 = 2r\pi(r + H)$
$P_2 = 2a\pi(a + b)$
$V_2 = BH = r^2H\pi$
$V_2 = ba^2\pi$

Односи:

$$\frac{P_1}{P_2} = \frac{2b\pi(a+b)}{2a\pi(a+b)} = \frac{b}{a}$$

$$\frac{V_1}{V_2} = \frac{ab^2\pi}{ba^2\pi} = \frac{b}{a}$$

16.



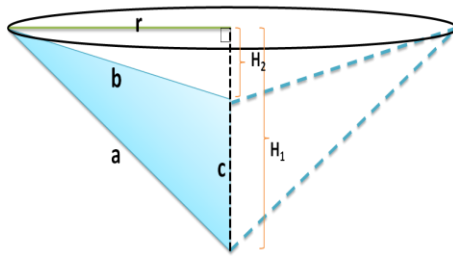
$P_{kvadrata} = a^2$
$72 = a^2 \Rightarrow a = 6\sqrt{2}$
$d = a\sqrt{2} = 12$

Две исте купе спојене заједничком основом:
–површина је збир два иста омотача
-запремина је збир запремина две исте купе

$r = \frac{d}{2} = \frac{12}{2} = 6$
$H = \frac{d}{2} = 6$
$s = a = 6\sqrt{2}$
$P_{tela} = 2M_{kupe} = 2rs\pi = 72\sqrt{2}cm^2$
$V_{tela} = 2V_{kupe} = 2 \cdot \frac{1}{3}r^2\pi H = 144\pi cm^3$

17. $a=15, b=13, c=4$

Око најмање странице:



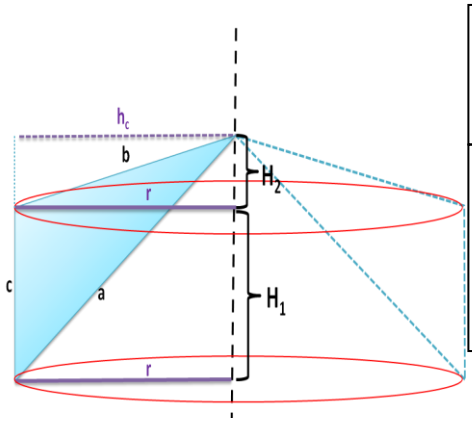
-купа из које је издубљена мања купа
-површина= збир омотача обе купа
-запремина=запремина веће-запремина мање купе

Већа купа:	Мања купа:
$r=h_c=12$	$r=h_c=12$
$s_1=a=15$	$s_2=b=13$
$H=H_1$	$H=H_2$
$M_{kv}=s_1r\pi=180\pi$	$M_{km}=s_2r\pi=156\pi$

$$P_1=M_{kv}+M_{km}=336\pi$$
$$V_1=V_{kv}-V_{km}=\frac{1}{3}r^2H_1\pi-\frac{1}{3}r^2H_2\pi$$
$$V_1=\frac{1}{3}r^2\pi\left(H_1-H_2\right)=\frac{1}{3}r^2\pi c=\frac{1}{3}144\cdot4\pi$$
$$V_1=192\pi$$

$$s_\Delta=\frac{15+13+4}{2}=16$$
$$P_\Delta=\sqrt{16(16-15)(16-13)(16-4)}$$
$$P_\Delta=\sqrt{16\cdot1\cdot3\cdot12}=24$$
$$P_\Delta=\frac{ch_c}{2}$$
$$24=\frac{4h_c}{2}\Rightarrow h_c=12$$

Око праве која садржи теме најмањег угла (угао наспрам најмање странице) и паралелна је наспрамној страници:



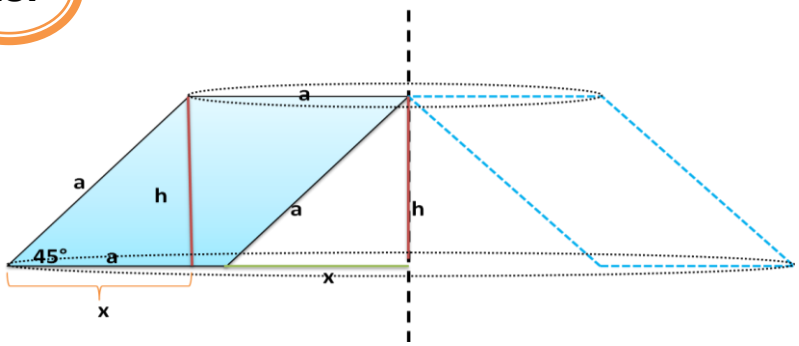
-ваљак и мања купа из којих је издубљена већа купа
-површина= збир омотача 2 купе и ваљка
-запремина=V ваљка+V мање купе-V веће купе

Ваљак:	Већа купа:	Мања купа:
$r=h_c=12$	$r=h_c=12$	$r=h_c=12$
$H=H_1=c=4$	$s_2=a=15$	$s_3=b=13$
$M_v=2rH\pi=96\pi$	$H=H_1+H_2$	$H=H_2$
	$M_{kv}=s_2r\pi=180\pi$	$M_{km}=s_3r\pi=156\pi$

$$P_2=M_v+M_{kv}+M_{km}=432\pi$$
$$V_2=V_v+V_{km}-V_{kv}=r^2H_1\pi+\frac{1}{3}r^2H_2\pi-\frac{1}{3}r^2\left(H_1+H_2\right)\pi$$
$$V_2=r^2H_1\pi+\frac{1}{3}r^2H_2\pi-\frac{1}{3}r^2H_1\pi-\frac{1}{3}r^2H_2\pi$$
$$V_2=\frac{2}{3}r^2H_1\pi=\frac{2}{3}144\cdot4\pi=384\pi cm^3$$

$$\frac{P_1}{P_2}=\frac{336\pi}{432\pi}=\frac{7}{9}$$
$$\frac{V_1}{V_2}=\frac{192\pi}{384\pi}=\frac{1}{2}$$

18.



$$a = 3$$
$$h^2 + x^2 = a^2$$
$$h = x \Rightarrow h^2 + h^2 = a^2$$
$$2h^2 = 9 \Rightarrow h = \frac{3}{\sqrt{2}} = x$$

Зарубљена купа из које је издубљена купа:
-површина=омотач зарубљене купе+омотач купе+горња база зарубљене купе +доња база зарубљене купе – база купе
-запремина=запремина зарубљене купе –запремина купе

Зарубљена купа

$$s = a = 3$$
$$H = h = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$
$$r_1 = a + x = 3 + \frac{3\sqrt{2}}{2} = \frac{6 + 3\sqrt{2}}{2}$$
$$r_2 = a = 3$$
$$M_{zk} = \left(r_1 + r_2\right)s\pi$$
$$M_{zk} = \left(\frac{12 + 3\sqrt{2}}{2}\right) \cdot 3\pi$$
$$M_{zk} = \frac{3\left(4 + \sqrt{2}\right)}{2} \cdot 3\pi$$
$$M_{zk} = \frac{9\left(4 + \sqrt{2}\right)\pi}{2}$$
$$B_1 = r_1^2\pi = \left(\frac{36 + 36\sqrt{2} + 18}{4}\right)\pi$$
$$B_1 = \frac{9\left(2\sqrt{2} + 3\right)}{2}\pi$$
$$B_2 = r_2^2\pi = 9\pi$$
$$V_{zk} = \frac{H\pi}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$
$$V_{zk} = \frac{\frac{3\sqrt{2}}{2}\pi}{3}\left(\frac{36 + 36\sqrt{2} + 18}{4} + \frac{18 + 9\sqrt{2}}{2} + 9\right)$$
$$V_{zk} = \frac{\sqrt{2}\pi}{2} \cdot \frac{36 + 36\sqrt{2} + 18 + 36 + 18\sqrt{2} + 36}{4}$$
$$V_{zk} = \frac{\sqrt{2}\pi}{2} \cdot \frac{126 + 54\sqrt{2}}{4} = \frac{\sqrt{2}\pi}{2} \cdot \frac{63 + 27\sqrt{2}}{2}$$
$$V_{zk} = \frac{\left(63\sqrt{2} + 54\right)\pi}{4}$$

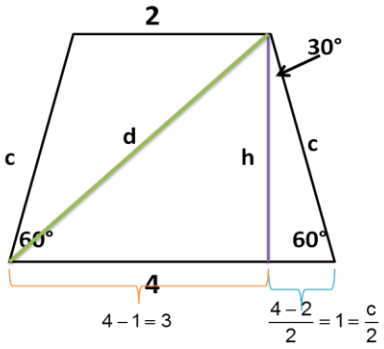
Купа

$$s = a = 3$$
$$H = h = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$
$$r_k = x = \frac{3\sqrt{2}}{2}$$
$$M_k = r_k s\pi$$
$$M_k = \frac{9\sqrt{2}\pi}{2}$$
$$B_k = r_k^2\pi = \frac{9\pi}{2}$$
$$V_k = \frac{1}{3}r_k^2 H\pi$$
$$V_k = \frac{1}{3} \cdot 9 \cdot \frac{3\sqrt{2}}{2}\pi$$
$$V_k = \frac{9\sqrt{2}\pi}{2}$$

$$P_{tela} = M_{zk} + M_k + B_2 + B_1 - B_k$$
$$P_{tela} = \frac{9\left(4 + \sqrt{2}\right)}{2}\pi + \frac{9\sqrt{2}}{2}\pi + 9\pi + \frac{9\left(2\sqrt{2} + 3\right)}{2}\pi - \frac{9}{2}\pi$$
$$P_{tela} = \pi\left(\frac{36 + 9\sqrt{2} + 9\sqrt{2} + 18 + 18\sqrt{2} + 27 - 9}{2}\right)$$
$$P_{tela} = \pi\frac{72 + 36\sqrt{2}}{2} = \left(36 + 18\sqrt{2}\right)\pi$$

$$V_{tela} = V_{zk} - V_k = \frac{\left(63\sqrt{2} + 54\right)\pi}{4} - \frac{9\sqrt{2}\pi}{2}$$
$$V_{tela} = \frac{\left(54\sqrt{2} + 54\right)\pi}{4} = \frac{27\left(\sqrt{2} + 1\right)}{2}\pi cm^3$$

19.



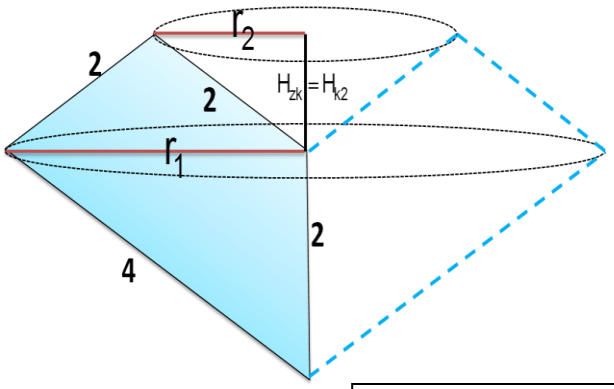
$c = 2 \cdot 1 = 2$

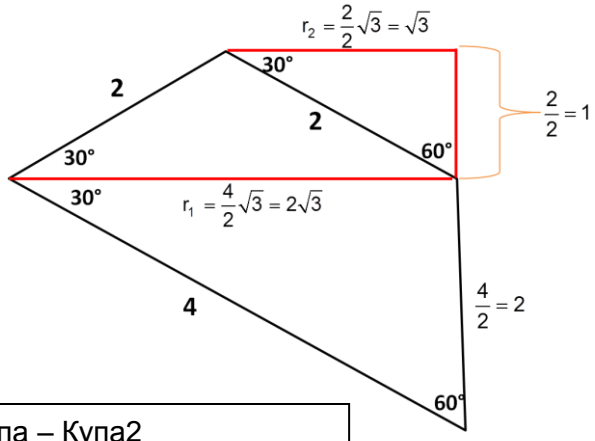
$h = \frac{c}{2}\sqrt{3} = \sqrt{3}$

$d^2 = 3^2 + \sqrt{3}^3 = 12$

$d = 2\sqrt{3}$

Око крака:





Тело:Купа1+Зарубљена купа – Купа2

Површина=$M_{K1}+M_{zk}+M_{K2}$

Запремина=$V_{K1}+V_{ZK}-V_{K2}$

Купа1:

$r = r_1 = 2\sqrt{3}$

$H = 2$

$s = 4$

$M_{K1} = rs\pi = 8\sqrt{3}\pi$

$V_{K1} = \frac{1}{3}r^2\pi H = 8\pi$

Зарубљена купа:

$r_1 = 2\sqrt{3}$

$r_2 = \sqrt{3}$

$H = 1$

$s = 2$

$M_{ZK} = (r_1 + r_2)s\pi = 6\sqrt{3}\pi$

$V_{ZK} = \frac{\pi H}{3}(r_1^2 + r_1 r_2 + r_2^2) = 7\pi$

Купа2:

$r = r_2 = \sqrt{3}$

$H = 1$

$s = 2$

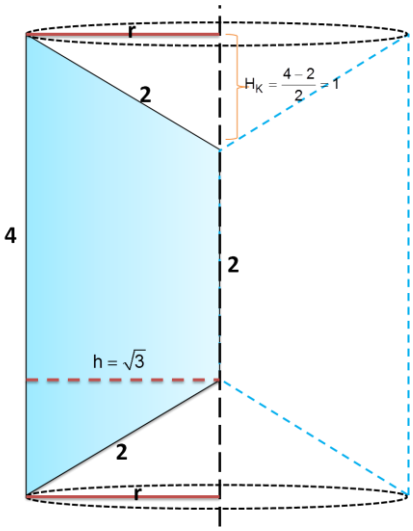
$M_{K2} = rs\pi = 2\sqrt{3}\pi$

$V_{K2} = \frac{1}{3}r^2\pi H = \pi$

$$P_{tela} = M_{K1} + M_{ZK} + M_{K2} = 16\sqrt{3}\pi$$

$$V_{tela} = V_{K1} + V_{ZK} - V_{K2} = 14\pi$$

Око мање основице:

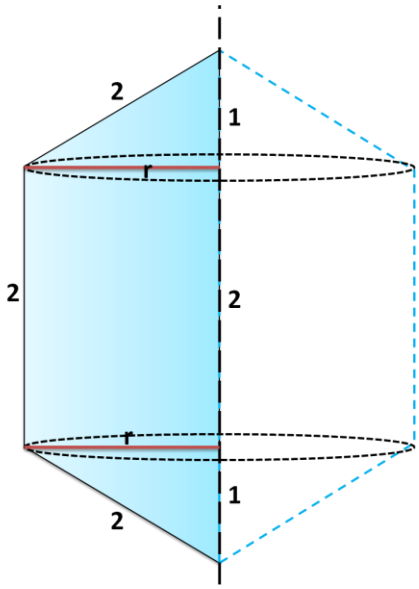


Тело:Ваљак из којег су издубљене две подударне купе
Површина=M_V+2M_K
Запремина=V_V-2V_K

Ваљак:	Купа:
$r = h = \sqrt{3}$ $H = 4$ $M_V = 2rH\pi = 8\sqrt{3}\pi$ $V_V = r^2\pi H = 12\pi$	$r = h = \sqrt{3}$ $H = 1$ $s = 2$ $M_K = rs\pi = 2\sqrt{3}\pi$ $V_K = \frac{1}{3}r^2\pi H = \pi$

$$P_{tela} = M_V + 2M_K = 12\sqrt{3}\pi$$
$$V_{tela} = V_V - 2V_K = 10\pi$$

Око веће основице:

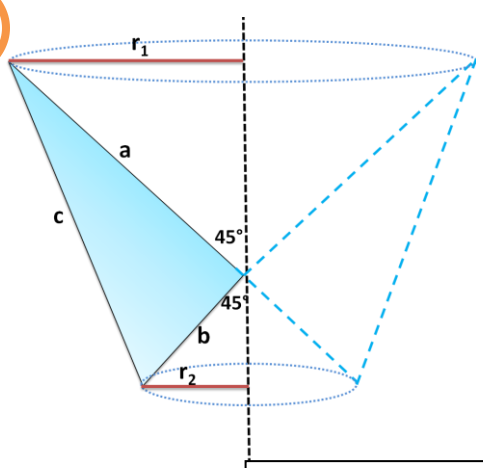


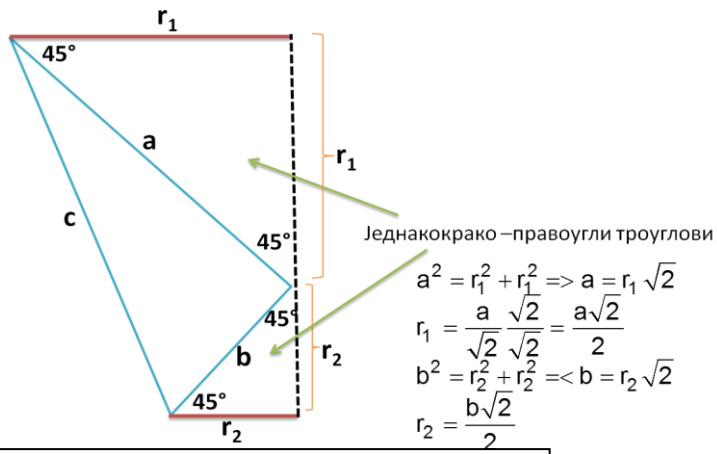
Тело:Ваљак + две подударне купе
Површина=M_V+2M_K
Запремина=V_V+2V_K

Ваљак:	Купа:
$r = h = \sqrt{3}$ $H = 2$ $M_V = 2rH\pi = 4\sqrt{3}\pi$ $V_V = r^2\pi H = 6\pi$	$r = h = \sqrt{3}$ $H = 1$ $s = 2$ $M_K = rs\pi = 2\sqrt{3}\pi$ $V_K = \frac{1}{3}r^2\pi H = \pi$

$$P_{tela} = M_V + 2M_K = 8\sqrt{3}\pi$$
$$V_{tela} = V_V + 2V_K = 8\pi$$

20.





$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Тело: Зарубљена купа- две различите купе

Површина=$M_{ZK}+M_{K1}+M_{K2}$

Запремина=$V_{ZK}-V_{K1}-V_{K2}$

Зарубљена купа:

$r_1 = \frac{a\sqrt{2}}{2}; r_2 = \frac{b\sqrt{2}}{2}$

$H = r_1 + r_2 = \frac{a\sqrt{2}}{2} + \frac{b\sqrt{2}}{2} = \frac{(a+b)\sqrt{2}}{2}$

$s = c = \sqrt{a^2 + b^2}$

$M_{ZK} = (r_1 + r_2)s\pi$

$M_{ZK} = \frac{(a+b)\sqrt{2}}{2}\sqrt{a^2 + b^2}\pi$

$V_{ZK} = \frac{H\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)$

$V_{ZK} = \frac{(a+b)\sqrt{2}\pi}{12}(a^2 + ab + b^2)$

Купа1:

$r = r_1 = \frac{a\sqrt{2}}{2}$

$H = r_1 = \frac{a\sqrt{2}}{2}$

$s = a$

$M_{K1} = rs\pi$

$M_{K2} = \frac{a\sqrt{2}}{2}a\pi = \frac{a^2\sqrt{2}}{2}\pi$

$V_{K1} = \frac{1}{3}r^2 H\pi$

$V_{K1} = \frac{1}{3}\cdot\frac{a^2}{2}\cdot\frac{a\sqrt{2}}{2}\pi = \frac{a^3\sqrt{2}}{12}\pi$

Купа1:

$r = r_2 = \frac{b\sqrt{2}}{2}$

$H = r_2 = \frac{b\sqrt{2}}{2}$

$s = b$

$M_{K2} = rs\pi$

$M_{K2} = \frac{b\sqrt{2}}{2}b\pi = \frac{b^2\sqrt{2}}{2}\pi$

$V_{K2} = \frac{1}{3}r^2 H\pi$

$V_{K2} = \frac{1}{3}\cdot\frac{b^2}{2}\cdot\frac{b\sqrt{2}}{2}\pi = \frac{b^3\sqrt{2}}{12}\pi$

$P_{tela} = M_{Zk} + M_{K1} + M_{K2}$

$P_{tela} = \frac{(a+b)\sqrt{2}}{2}\sqrt{a^2 + b^2}\pi + \frac{a^2\sqrt{2}}{2}\pi + \frac{b^2\sqrt{2}}{2}\pi$

$P_{tela} = \frac{\pi\sqrt{2}}{2}\left((a+b)\sqrt{a^2 + b^2} + a^2 + b^2\right)$

$V_{tela} = V_{ZK} - V_{K1} - V_{K2}$

$V_{tela} = \frac{(a+b)\sqrt{2}\pi}{12}(a^2 + ab + b^2) - \frac{a^3\sqrt{2}\pi}{12} - \frac{b^3\sqrt{2}\pi}{12}$

$v_{tela} = \frac{\pi\sqrt{2}}{12}\left((a+b)(a^2 + ab + b^2) - a^3 - b^3\right)$

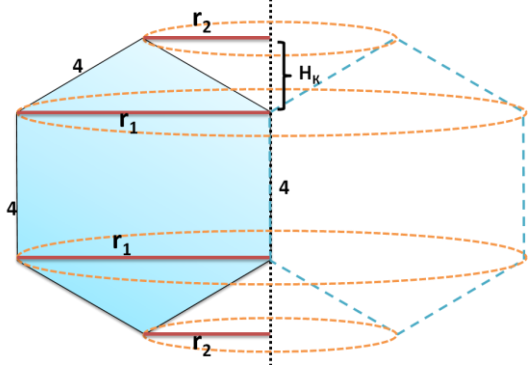
$V_{tela} = \frac{\pi\sqrt{2}}{12}(2a^2 b + 2ab^2) = \frac{2ab\pi\sqrt{2}}{12}(a+b) = \frac{ab\pi\sqrt{2}}{6}(a+b)$

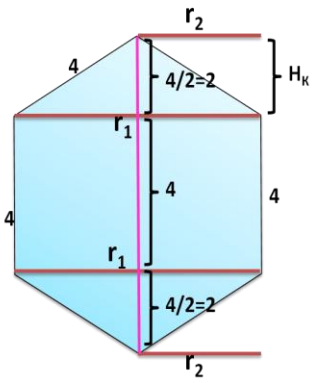
21.

$$P_{š} = \frac{3a^2\sqrt{3}}{2} = 24\sqrt{3}$$

$$\frac{3a^2\sqrt{3}}{2} = 24\sqrt{3} \Rightarrow \frac{3a^2}{2} = 24 \Rightarrow a^2 = 16$$

$$a = 4$$





r_1-мања дијагонала шестоугла

r_2-половина мање дијагонале шестоу

$$d_m = a\sqrt{3} = 4\sqrt{3}$$

$$r_1 = 4\sqrt{3}$$

$$r_2 = 2\sqrt{3}$$

Тело: Ваљак+ две подударне зарубљене купе-две подударне купе

Површина=$M_V+2M_{ZK}+2M_K$

Запремина=$V_V+2V_{ZK}-2V_K$

Ваљак:

$r = r_1 = 4\sqrt{3}$

$H = 4$

$M_V = 2rH\pi = 32\sqrt{3}\pi$

$V_V = r^2H\pi = 192\pi$

Зарубљена купа:

$r_1 = 4\sqrt{3}; r_2 = 2\sqrt{3}$

$H = 2$

$s = 4$

$M_{ZK} = (r_1 + r_2)s\pi = 24\sqrt{3}\pi$

$V_{ZK} = \frac{H\pi}{3}(r_1^2 + r_1r_2 + r_2^2) = 56\pi$

Купа:

$r = r_2 = 2\sqrt{3}$

$H = 2$

$s = 4$

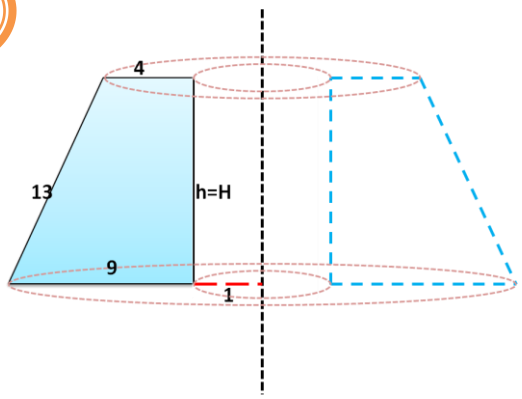
$M_K = rs\pi = 8\sqrt{3}\pi$

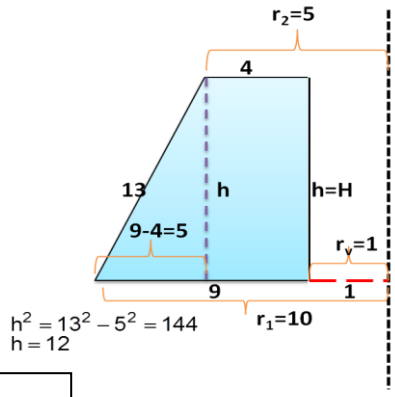
$V_K = \frac{1}{3}r^2H\pi = 8\pi$

$$P_{tela} = M_V + 2M_{ZK} + 2M_K = 96\sqrt{3}\pi$$

$$V_{tela} = V_V + 2(V_{ZK} - V_K) = 288\pi$$

22.





$h^2 = 13^2 - 5^2 = 144$
$h = 12$

Тело: Зарубљена купа – ваљак
Површина=$M_{ZK}+M_V+B_{ZK1}+B_{ZK2}-2B_V$
Запремина=$V_{ZK}-V_V$

Зарубљена купа:

$r_1 = 10; r_2 = 5$
$H = 12$
$s = 13$
$M_{ZK} = (r_1 + r_2)s\pi = 195\pi$
$B_{ZK1} = r_1^2\pi = 100\pi$
$B_{ZK2} = r_2^2\pi = 25\pi$
$V_{ZK} = \frac{H\pi}{3}(r_1^2 + r_1 r_2 + r_2^2) = 700\pi$

Ваљак:

$r = 1$
$H = 12$
$M_V = 2rH\pi = 24\pi$
$B_V = r^2\pi = \pi$
$V_V = r^2H\pi = 12\pi$

$$P_{tela} = M_{ZK} + B_{ZK1} + B_{ZK2} + M_V - 2B_V$$
$$P_{tela} = 342\pi$$
$$V_{tela} = V_{ZK} - V_V = 688\pi$$