1.

$R_1 = R + 1$

$P_1 = P + 8\pi$

$P = 4R^2\pi$

$P_1 = 4R_1^2\pi = 4(R+1)^2\pi$

$4(R+1)^2\pi = P + 8\pi$

$4(R+1)^2\pi = 4R^2\pi + 8\pi \quad /:4\pi$

$(R+1)^2 = R^2 + 2$

$R^2 + 2 = R^2 + 2R + 1$

$2R = 1 \Rightarrow R = \frac{1}{2}$

$R_1 = R + 1 \Rightarrow R_1 = \frac{3}{2}$

$$V_1 - V = \frac{4}{3}R_1^3\pi - \frac{4}{3}R^3\pi =$$

$$= \frac{4}{3}\pi\left(\left(\frac{3}{2}\right)^3 - \left(\frac{1}{2}\right)^3\right) = \frac{4}{3}\pi \cdot \frac{26}{8}$$

$$= \frac{13}{3}\pi$$

2.

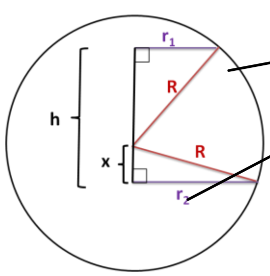
$h = 7$

$r_1 = 16$

$r_2 = 33$

$P_Z = 2R\pi h \Rightarrow R = ?$

Претпоставимо да је лопта пресечена са различитих страна центра:



$R^2 = (h-x)^2 + r_1^2$

$R^2 = x^2 + r_2^2$

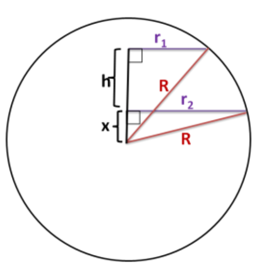
$(h-x)^2 + r_1^2 = x^2 + r_2^2$

$(7-x)^2 + 256 = x^2 + 1089$

$49 - 14x + x^2 + 256 = x^2 + 1089$

$-14x = 784 \Rightarrow x = -56 < 0$

=> лопта је пресечена са исте стране центра (=> x=+56)



$R^2 = h^2 + r_1^2$

$R^2 = (h+x)^2 + r_2^2$

$(h+x)^2 + r_1^2 = x^2 + r_2^2$

$(7+x)^2 + 256 = x^2 + 1089$

$49 + 14x + x^2 + 256 = x^2 + 1089$

$14x = 784 \Rightarrow x = 56$

$R^2 = x^2 + r_2^2$

$R^2 = 4225 \Rightarrow R = 65$

$P_{Zone} = 2 \cdot 65 \cdot \pi \cdot 7 = 910\pi$

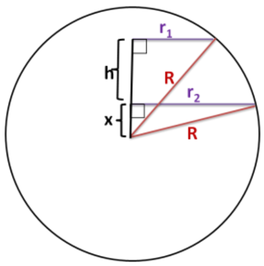
3.

$h = 3$

$r_1 = 9$

$r_2 = 12$

$P = 4R^2\pi \Rightarrow R = ?$



$(h+x)^2 + r_1^2 = x^2 + r_2^2$

$(3+x)^2 + 81 = x^2 + 144$

$9 + 6x + x^2 + 81 = x^2 + 144$

$6x = 54$

$x = 9$

$R^2 = x^2 + r_2^2$

$R^2 = 81 + 144 = 225$

$P = 4 \cdot 225 \cdot \pi = 900\pi$

4.

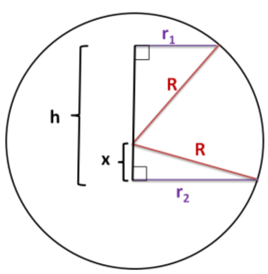
$R = 65$

$r_1 = 25$

$r_2 = 33$

$P_{Pojasa} = 2R\pi h \Rightarrow h?$

***ако је лопта пресечена са различитих страна центра**



$R^2 = x^2 + r_2^2$

$4225 = x^2 + 1089 \Rightarrow x^2 = 3136 \Rightarrow x = 56$

$R^2 = (h-x)^2 + r_1^2$

$4225 = (h-56)^2 + 625$

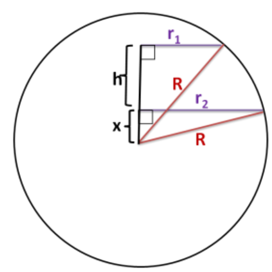
$(h-56)^2 = 3600$

$h - 56 = 60$

$h = 116$

$P_{Pojasa} = 2 \cdot 65\pi \cdot 116 = 15080\pi$

***ако је лопта пресечена са исте стране центра**



$R^2 = x^2 + r_2^2$

$4225 = x^2 + 1089 \Rightarrow x^2 = 3136 \Rightarrow x = 56$

$R^2 = (h+x)^2 + r_1^2$

$4225 = (h+56)^2 + 625$

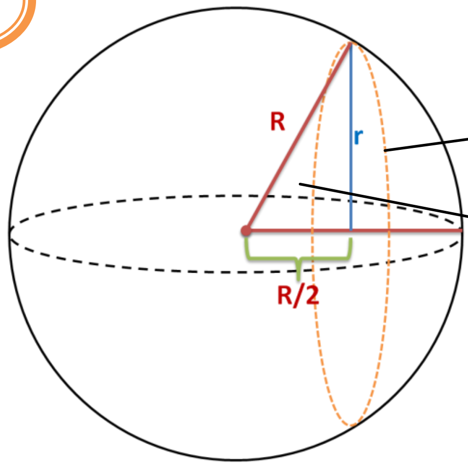
$(h+56)^2 = 3600$

$h + 56 = 60$

$h = 4$

$P_{Pojasa} = 2 \cdot 65\pi \cdot 4 = 520\pi$

5.



$$P_{preseka} = 48\pi$$

$$P_{preseka} = r^2\pi = 48\pi \Rightarrow r^2 = 48$$

$$R^2 = \left(\frac{R}{2}\right)^2 + r^2$$

$$R^2 - \frac{R^2}{4} = 48 \Rightarrow \frac{3R^2}{4} = 48 \Rightarrow R^2 = 64 \Rightarrow R = 8cm$$

6.

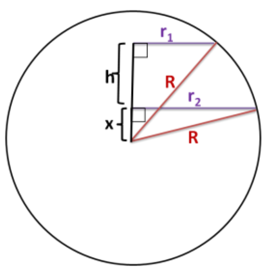
$$2R = 24 \Rightarrow R = 12$$

$$P_1 = r_1^2\pi = 135\pi \Rightarrow r_1^2 = 135$$

$$P_2 = r_2^2\pi = 140\pi \Rightarrow r_2^2 = 140$$

$$h = ?$$

***ако је пресечена са исте стране центра**



$$R^2 = x^2 + r_2^2$$

$$144 = x^2 + 140 \Rightarrow x^2 = 4 \Rightarrow x = 2$$

$$R^2 = (h + x)^2 + r_1^2$$

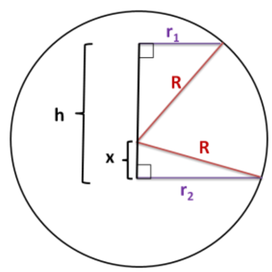
$$144 = (h + 2)^2 + 135$$

$$(h + 2)^2 = 9$$

$$h + 2 = 3$$

$$h = 1cm$$

***ако је пресечена са различитих страна центра**



$$R^2 = x^2 + r_2^2$$

$$144 = x^2 + 140 \Rightarrow x^2 = 4 \Rightarrow x = 2$$

$$R^2 = (h - x)^2 + r_1^2$$

$$144 = (h - 2)^2 + 135$$

$$(h - 2)^2 = 9$$

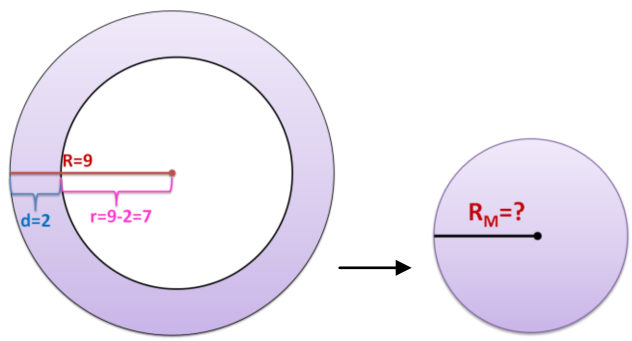
$$h - 2 = 3$$

$$h = 5cm$$

7.

Запремина шупље лопте = запремина масивне лопте
Спољашњи пречник 2R=18 => спољашњи полупречник R=9
Унутрашњи полупречник: r=9-2=7

Пресек:



Запремина шупље лопте:

$$\frac{4}{3}R^3\pi - \frac{4}{3}r^3\pi = \frac{4}{3}\pi\left(9^3 - 7^3\right) = \frac{4}{3}\cdot 386\pi$$

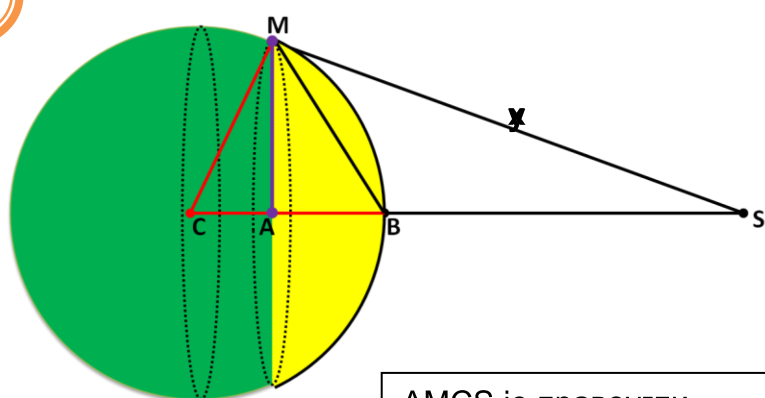
Запремина масивне лопте:

$$V_M = \frac{4}{3}R_M^{\ 3}\pi$$

$$\frac{4}{3}386\pi = \frac{4}{3}R_M^3 \Rightarrow R_M^3 = 386$$

$$R_M = \sqrt[3]{386}$$

8.



$CM = CB = R$
$MA = r$
$AB = h$ – висина калоте
BS=x;MS=y
$CS = R + x = ?$

$$P_{kalota} = \frac{1}{3}P$$

$$2R\pi h = \frac{1}{3}4R^2\pi \Rightarrow h = \frac{2}{3}R$$

ΔMAC – правоугли:

$$|MA|^2 = |MC|^2 - |CA|^2$$

$$r^2 = R^2 - (R-h)^2$$

$$r^2 = R^2 - \left(R - \frac{2}{3}R\right)^2$$

$$r^2 = R^2 - \left(\frac{1}{3}R\right)^2 = R^2 - \frac{8}{9}R^2$$

$$r^2 = \frac{8}{9}R^2 \Rightarrow r = \frac{2\sqrt{2}}{3}R$$

ΔMCS је правоугли
(угао између тангенте и полупречника у тачки додира је прав)

$$|MS|^2 = |CS|^2 - |CM|^2$$

$$y^2 = (R+x)^2 - R^2$$

ΔMAS -правоугли

$$y^2 = r^2 + (x+h)^2$$

$$y^2 = \frac{8}{9}R^2 + \left(x + \frac{2}{3}R\right)^2$$

$$(R+x)^2 - R^2 = \frac{8}{9}R^2 + \left(x + \frac{2}{3}R\right)^2$$

$$R^2 + 2Rx + x^2 - R^2 = \frac{8}{9}R^2 + x^2 + \frac{4}{3}xR + \frac{4}{9}R^2$$

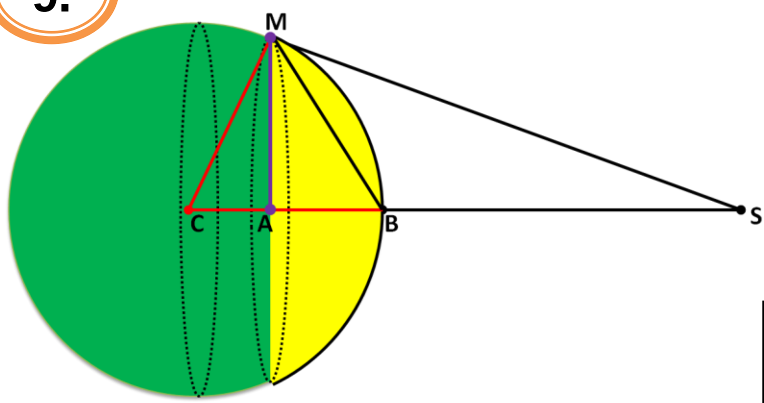
$$2Rx - \frac{4}{3}Rx = \frac{8}{9}R^2 + \frac{4}{9}R^2$$

$$\frac{2}{3}Rx = \frac{12}{9}R^2 \quad /:R \Rightarrow \frac{2}{3}x = \frac{4}{3}R \Rightarrow \underline{\underline{x = 2R}}$$

Тражено растојање:

$$CS = R + x = R + 2R = \underline{\underline{3R}}$$

9.



$|CM| = |CB| = 2$

$|CS| = 4$

$x = |BS| = |CS| - |SM| = 4 - 2 =$

$MA = r$

$AB = h -$ висина калоте

$P_{kalota} = 2Rh\pi$

ΔMCS: $|MS|^2 = |CS|^2 - R^2 = 16 - 4 = 12$

$MS = 2\sqrt{3}$

$P_{\Delta MCS} = \frac{|MC| \cdot |MS|}{2} = \frac{|CS| \cdot |MA|}{2} \Rightarrow$

$|MC| \cdot |MS| = |CS| \cdot |MA|$

$R \cdot y = |CS| \cdot r \Rightarrow r = \frac{Ry}{|CS|} = \frac{4\sqrt{3}}{4} = \sqrt{3}$

ΔMAS:

$|MS|^2 = r^2 + (x+h)^2$

$12 = 3 + (2+h)$

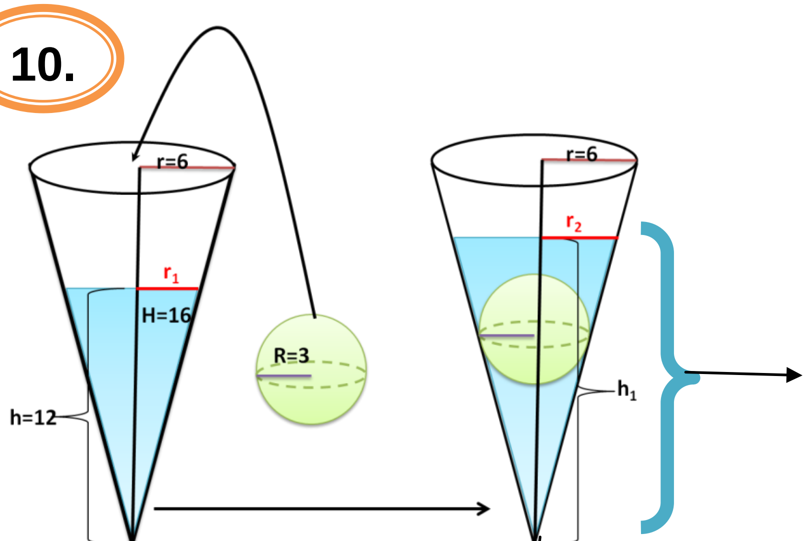
$(2+h)^2 = 3$

$2 + h = 3 \Rightarrow h = 1$

Површина осветљеног дела лопте (површина калоте):

$P_{Kalote} = 2R\pi h = 2 \cdot 2\pi \cdot 1 = 4\pi$

10.



Запремина воде у купи:

$V_V = \frac{1}{3} r_1^2 \pi H = \frac{81}{12} \pi \cdot 12 = 81\pi$

Запремина лопте:

$V_L = \frac{4}{3} R^3 \pi = 36\pi$

Запремина ,купе' са водомо

$V = V_V + V_L = 117\pi$

Из сличности троуглова:

$12 : r_1 = 16 : 6$

$r_1 = \frac{12 \cdot 6}{16} = \frac{9}{2}$

$r_2 : h_1 = 6 : 16$

$r_2 = \frac{6h_1}{16} = \frac{3h_1}{8}$

$\Rightarrow$

$V = \frac{1}{3} r_2^2 h_1 \pi = 117\pi$

$r_2^2 h_1 = 3 \cdot 117$

$\frac{9h_1^2}{64} h_1 = 3 \cdot 117$

$h_1^3 = \frac{64 \cdot 3 \cdot 117}{9}$

$h_1^3 = \frac{4^3 \cdot 117}{3} = 4^3 \cdot 39 \quad / \sqrt[3]{\ }$

$h_1 = 4\sqrt[3]{39}$