**BINOMNA (Njutnova) FORMULA**

\[
(x + y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1} y^1 + \binom{n}{2}x^{n-2} y^2 + \ldots + \binom{n}{k}x^{n-k} y^k + \ldots + \binom{n}{n}x^0 y^n
\]

\[
\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n} - \text{BINOMNI KOEFFICIJENTI}
\]

(k+1)-član u razvoju binoma: \( T_{k+1} = \binom{n}{k} x^{n-k} y^k \)

**OSOBINE BINOMNIH KOEFFICIJENATA:**

1. \( \binom{n}{k} = \binom{n}{n-k} \)
2. \( \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k} \)
3. \( \binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k} = \binom{n-m+k}{k} \)

**PASKALOV TROUGAO**

- binomni koeficijenti u razvoju \((x + y)^n\)

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**ZADACI:**

1. **Razviti binom:**
   a) \((x + 2)^6\)

   **rešenje:**
   \[
   (x + 2)^6 = \binom{6}{0}x^6 + \binom{6}{1}x^5 \cdot 2^1 + \binom{6}{2}x^4 \cdot 2^2 + \binom{6}{3}x^3 \cdot 2^3 + \binom{6}{4}x^2 \cdot 2^4 + \binom{6}{5}x^1 \cdot 2^5 + \binom{6}{6} \cdot 2^6 = \\
   = 1 \cdot x^6 + 6 \cdot x^5 \cdot 2^1 + 6 \cdot 5 \cdot x^4 \cdot 2^2 + 6 \cdot 5 \cdot 4 \cdot x^3 \cdot 2^3 + 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2^4 + 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot x^2 \cdot 2^5 + 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2^6 \cdot x^1 = \\
   = x^6 + 6 \cdot x^5 + 60 \cdot x^4 + 160 \cdot x^3 + 240 \cdot x^2 + 192 \cdot x + 64
   \]

   b) \((x - 1)^5\)

   **rešenje:**
   \[
   (x - 1)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4(-1)^1 + \binom{5}{2}x^3(-1)^2 + \binom{5}{3}x^2(-1)^3 + \binom{5}{4}x^1(-1)^4 + \binom{5}{5}(-1)^5 = \\
   = 1 \cdot x^5 + 5 \cdot x^4(-1)^1 + 5 \cdot 4 \cdot x^3(-1)^2 + 5 \cdot 4 \cdot 3 \cdot x^2(-1)^3 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot x^1(-1)^4 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot (-1)^5 = \\
   = x^5 - 5 \cdot x^4 + 10 \cdot x^3 - 10 \cdot x^2 + 5 \cdot x - 1
   \]
c) \((2x + y)^4\)

Rešenje:
\[
(2x + y)^4 = \binom{4}{0}(2x)^4 + \binom{4}{1}(2x)^3y + \binom{4}{2}(2x)^2y^2 + \binom{4}{3}(2x)^1y^3 + \binom{4}{4}y^4 =
\]
\[
= 16x^4 + 4 \cdot 8x^3y + \frac{4 \cdot 3}{2 \cdot 1} 4x^2y^2 + \binom{4}{1}2xy^3 + \binom{4}{0}y^4 =
\]
\[
= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4
\]

2. Odrediti:

a) četvrti član u razvoju binoma \((x^2 - y^2)^{11}\)

Rešenje:
\[
T_4 = \binom{11}{3}x^8(-y)^3 = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} x^8(-y)^3 = -165x^8y^3
\]

b) šesti član u razvoju binoma \((x + y)^{15}\)

Rešenje:
\[
T_6 = \binom{15}{5}x^{10}y^5 = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} x^{10}y^5 = 3003x^{10}y^5
\]

c) peti član u razvoju binoma \((\sqrt{x} - \sqrt{y})^{12}\)

Rešenje:
\[
T_5 = \binom{12}{4}(\sqrt{x})^4(-\sqrt{y})^4 = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} x^4y^2 = 495x^4y^2
\]

d) binomni koeficijent uz peti član u razvoju binoma \((2\sqrt{x} - 1)^8\)

Rešenje:
\[
\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70
\]

Binomni koeficijent uz peti član je 70.

e) koeficijent uz peti član u razvoju binoma \((2\sqrt{x} - 1)^8\)

Rešenje:
\[
T_5 = \binom{8}{4}(2\sqrt{x})^4(-1)^4 = 70 \cdot 16x^2 \cdot 1 = 1120x^2
\]

Koeficijent uz peti član je 1120.
3. U razvoju sledećih binoma odrediti član koji ne sadrži x

a) \( \left( \frac{4a^2x}{\sqrt[3]{ax^2}} \right)^{13} \)

rešenje:

\[
T_{k+1} = \binom{13}{k} \left( \frac{4a^2x}{\sqrt[3]{ax^2}} \right)^{13-k} \left( \frac{1}{\sqrt[3]{ax^2}} \right)^k
= \binom{13}{k} \left( \frac{1}{a^2 \cdot x^4} \right)^{13-k} \left( \frac{-1}{a^5 \cdot x^{-5}} \right)^k
= \binom{13}{k} a^{13-k} \cdot x^{13-k} \cdot \frac{1}{a^{13} \cdot x^{13-k}}
\]

Za član koji ne sadrži x važi:

\[
x^0 = \frac{13-k}{4} \cdot x^4 + \frac{-2k}{5} = 0 / \cdot 20
\]

\[
5(13 - k) - 8k = 0
\]

\[
65 - 5k - 8k = 0
\]

\[
65 = 13k
\]

\[
k = 5
\]

Član koji ne sadrži x je 6. član:

\[
T_6 = \binom{13}{5} \left( \frac{4a^2x}{\sqrt[3]{ax^2}} \right)^5 \left( \frac{1}{\sqrt[3]{ax^2}} \right)^5
= \binom{13}{5} \left( \frac{1}{a^{2 \cdot x^4}} \right)^8 \left( \frac{-1}{a^5 \cdot x^{-5}} \right)^5
= \binom{13}{5} a^4 \cdot x^2 \cdot a^{-1} \cdot x^{-2} = 1287a^3
\]

b) \( \left( \frac{3\sqrt[3]{x}}{\sqrt{x^{-1}}} \right)^{15} \)

rešenje:

\[
T_{k+1} = \binom{15}{k} \left( \frac{3\sqrt[3]{x}}{\sqrt{x^{-1}}} \right)^{15-k} \left( \sqrt{x^{-1}} \right)^k
= \binom{15}{k} \left( \frac{1}{x^3} \right)^{15-k} \left( \frac{-1}{x} \right)^k
= \binom{15}{k} x^{15-k} \cdot \frac{-k}{3} \cdot x^2
\]

Za član koji ne sadrži x važi:

\[
x^0 = \frac{15-k}{3} \cdot x^3 + \frac{-k}{2} = 0 / \cdot 6
\]

\[
2(15 - k) - 3k = 0
\]

\[
30 - 2k - 3k = 0
\]

\[
30 = 5k
\]

\[
k = 6
\]

Član koji ne sadrži x je 7. član:

\[
T_7 = \binom{15}{6} \left( \frac{3\sqrt[3]{x}}{\sqrt{x^{-1}}} \right)^{15-6} \left( \sqrt{x^{-1}} \right)^6
= \binom{15}{6} \left( \frac{1}{x^3} \right)^9 \left( \frac{-1}{x} \right)^6
= \binom{15}{6} x^3 \cdot x^{-3} = \binom{15}{6}
\]
c) \( \left( x^2 + \frac{1}{x} \right)^n \) ako je zbir koeficijenata prva tri člana jednak 46

rešenje:
U ovom slučaju binomni koeficijent = koeficijent
\[
\binom{n}{0} + \binom{n}{1} + \binom{n}{2} = 46
\]
\[
1 + n + \frac{n(n-1)}{2} = 46 \div 2
\]
\[
2 + 2n + n^2 - n = 92
\]
\[
n^2 + n = 90
\]
\[
n_1 = -10 (\perp \text{ jer } n \in \mathbb{N})
\]
\[
n_2 = 9
\]
\[
\left( x^2 + \frac{1}{x} \right)^n = \left( x^2 + \frac{1}{x} \right)^9
\]
\[
T_{k+1} = \binom{9}{k} x^{9-k} \left( \frac{1}{x} \right)^k = \binom{9}{k} x^{18-2k} x^{-k}
\]
Za član koji ne sadrži x važi:
\[
x^{18-2k} \cdot x^{-k} = x^0
\]
\[
18 - 2k - k = 0
\]
\[
18 - 3k = 0
\]
\[
18 = 3k
\]
\[
k = 6
\]
Član koji ne sadrži x je 7. član: 
\[
T_7 = \binom{9}{6} x^{9-6} \left( \frac{1}{x} \right)^6 = \binom{9}{3} x^6 \frac{1}{x^6} = \binom{9}{3} = 84
\]

d) \( \left( \sqrt[3]{x^2} + \frac{y}{x} \right)^n \) ako je binomni koeficijent trećeg člana za 5 veći od binomnog koeficijenta drugog člana

rešenje:
\[
\binom{n}{2} = \binom{n}{1} + 5
\]
\[
\frac{n(n-1)}{2} = n + 5 \div 2
\]
\[
n^2 - n = 2n + 10
\]
\[
n^2 - 3n - 10 = 0
\]
\[
n_1 = -2 (\perp \text{ jer } n \in \mathbb{N})
\]
\[
n_2 = 5
\]
\[
\left(\frac{3\sqrt{x^2} + y}{x}\right)^n = \left(\frac{3\sqrt{x^2} + y}{x}\right)^5
\]

\[T_{k+1} = \binom{5}{k} \left(\frac{3\sqrt{x^2}}{x}\right)^{5-k} \left(\frac{y}{x}\right)^k = \binom{5}{k} \left(\frac{2}{x^3}\right)^{5-k} \cdot y^k \cdot x^{-k} = \binom{5}{k} \cdot \frac{10-2k}{3} \cdot y^k \cdot x^{-k}
\]

Za član koji ne sadrži x važi:
\[
x^{\frac{10-2k}{3}} \cdot x^{-k} = x^0
\]
\[
\frac{10-2k}{3} - k = 0 \quad / \cdot 3
\]
\[
10 - 2k - 3k = 0
\]
\[
10 = 5k
\]
\[
k = 2
\]

Član koji ne sadrži x je 3. član:
\[
T_3 = \binom{5}{2} \left(\frac{3\sqrt{x^2}}{x}\right)^{5-2} \left(\frac{y}{x}\right)^2 = \binom{5}{2} \left(\frac{2}{x^3}\right)^3 \cdot \frac{y^2}{x^2} = 10x^2 \cdot \frac{y^2}{x^2} = 10y^2
\]

\[
e) \left(\frac{x^2 + a}{x}\right)^n \text{ ako su binomni koeficijenti četvrtog i trinaestog člana jednaki}
\]

rešenje:
\[
\binom{n}{3} = \binom{n}{12} \Rightarrow (k = 3 \land n - k = 12) \Rightarrow n - 3 = 12 \Rightarrow n = 15
\]
\[
\left(\frac{x^2 + a}{x}\right)^n = \left(\frac{x^2 + a}{x}\right)^{15}
\]
\[
T_{k+1} = \binom{15}{k} \left(\frac{x^3}{x}\right)^{15-k} \left(\frac{a}{x}\right)^k = \binom{15}{k} \cdot x^{30-2k} \cdot a^k \cdot x^{-k}
\]

Za član koji ne sadrži x važi:
\[
x^{30-2k} \cdot x^{-k} = x^0
\]
\[
30 - 2k - k = 0
\]
\[
30 - 3k = 0
\]
\[
30 = 3k
\]
\[
k = 10
\]

Član koji ne sadrži x je 11. član:
\[
T_{11} = \binom{15}{10} \left(\frac{x^3}{x}\right)^{15-10} \left(\frac{a}{x}\right)^{10} = \binom{15}{5} \cdot \frac{a^{10}}{x^{10}} = 3003a^{10}
\]
f) \( \left( \sqrt[3]{x} + \frac{1}{3\sqrt{x^2}} \right)^n \) ako se koeficijenti petog i trećeg člana odnose kao 7:2

rešenje:

\[
\begin{align*}
\binom{n}{4} : \binom{n}{2} &= 7 : 2 \\
n(n-1)(n-2)(n-3) : \frac{n(n-1)}{2} &= 7 : 2 \\
2 \frac{n(n-1)(n-2)(n-3)}{4 \cdot 3 \cdot 2 \cdot 1} &= 7 \frac{n(n-1)}{2} / : n(n-1) \\
\frac{(n-2)(n-3)}{4 \cdot 3} &= \frac{7}{2} \cdot \frac{1}{12} \\
(n-2)(n-3) &= 7 \cdot 6 \\
n-2 &= 7 \Rightarrow n = 9 \\
\left( \sqrt[3]{x} + \frac{1}{3\sqrt{x^2}} \right)^n &= \left( \sqrt[3]{x} + \frac{1}{3\sqrt{x^2}} \right)^9 \\
T_{k+1} &= \binom{9}{k} \left( \sqrt[3]{x} \right)^{9-k} \left( \frac{1}{3\sqrt{x^2}} \right)^k = \binom{9}{k} \left( \frac{1}{x} \right)^{3-k} \cdot \left( \frac{-2}{3} \right)^k = \binom{9}{k} \frac{9-k}{2} \cdot \frac{-2k}{3}
\end{align*}
\]

Za član koji ne sadrži \( x \) važi:

\[
\begin{align*}
&x^{\frac{9-k}{2}} \cdot \frac{-2k}{3} = x^0 \\
&\frac{9-k}{2} + \frac{-2k}{3} = 0 / : 6 \\
&27 - 3k - 4k = 0 \\
&27 = 7k \\
&k = \frac{27}{7} \quad (\perp \text{jer } k \in \mathbb{N})
\end{align*}
\]

Ne postoji član koji ne sadrži \( x \).

4. Koji član u razvoju binoma \( \left( \frac{3}{\sqrt[3]{a}} + \frac{1}{\sqrt[3]{b}} \right)^{21} \) sadrži \( a \) i \( b \) sa istim stepenom?

\[
\begin{align*}
T_{k+1} &= \binom{21}{k} \left( \frac{a^{1/3} \cdot b^{-1/6}}{3\sqrt[3]{a}} \right)^{21-k} \left( \frac{b^{1/2} a^{-1/6}}{3\sqrt[3]{a}} \right)^k = \binom{21}{k} \frac{21-k}{3} \cdot \frac{-21+k}{6} \cdot \frac{k}{b^{1/2} a^{1/6}} = \binom{21}{k} \frac{21-k}{3} \cdot \frac{-21+k}{6} \cdot \frac{k}{b^{1/2} a^{1/6}} \\
\text{Ako } a \text{ i } b \text{ imaju isti eksponent: } &\quad a^{\frac{21-k}{3}} \cdot a^{\frac{-k}{6}} = b^{\frac{-21+k}{6}} \cdot b^{\frac{k}{2}} \cdot a^{\frac{-k}{6}}
\end{align*}
\]
\[
\frac{21-k}{3} + \frac{-2k}{6} = \frac{-21+k}{6} + \frac{k}{2} \div 6
\]
\[
42 - 2k - 2k = -21 + k + 3k
\]
\[
63 = 7k
\]
\[
k = 9
\]

Deseti član.

5. Odrediti:

a) srednji član u razvoju binoma \(a^{-2}\sqrt{a} - \frac{5}{\sqrt{a}}\) ako se koeficijenti petog i trećeg člana odnose kao 14:3

rešenje:

\[
\left(a^{-2}\sqrt{a} - \frac{5}{\sqrt{a}}\right)^m = \left(a^{-2} \cdot a^{1/2} - (a^{-2} \cdot a^{-1/2})^{1/5}\right)^m = \left(a^{-3/2} - (a^{-5/2})^{1/5}\right)^m = \left(a^{-3} - a^{-1}\right)^m
\]

\[
T_5 = \binom{m}{4}(a^{-3/2})^m \cdot (-a^{-1/2})^4 = \binom{m}{4} a^{-3m+12} a^{-4} = \binom{m}{4} a^{-2} a^{-4} \Rightarrow \text{koeficijent 5. člana je } \binom{m}{4}
\]

\[
T_3 = \binom{m}{2}(a^{-3/2})^m \cdot (-a^{-1/2})^2 = \binom{m}{2} a^{-3m+6} a^{-2} \Rightarrow \text{koeficijent 3. člana je } \binom{m}{2}
\]

\[
\binom{m}{4} : \binom{m}{2} = 14:3
\]

\[
m(m-1)(m-2)(m-3) \cdot \frac{m(m-1)}{4 \cdot 3 \cdot 2 \cdot 1} = 14 : 3
\]

\[
m(m-1)(m-2)(m-3) \cdot \frac{2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 14 \frac{m(m-1)}{2 \cdot 1} \div m(m-1)
\]

\[
(m-2)(m-3) = 7
\]

\[
(m-2)(m-3) = 8 \cdot 7
\]

\[
m - 2 = 8 \Rightarrow m = 10
\]

Za m=10 binom ima 11 članova; srednji član je 6. član (k=5):

\[
T_6 = \binom{10}{5}(a^{-3/2})^5 (-a^{-1/2})^5 = \binom{10}{5} a^{-15} a^{-5} = \binom{10}{5} a^{-20} = \binom{10}{5} a^{-10} = \binom{10}{5} a^{10}
\]
b) peti član u razvoju binoma \((\sqrt{1+x} - \sqrt{1-x})^n\) ako je koeficijent trećeg člana jednak 78

rešenje:

\[
T_3 = \binom{n}{2}((\sqrt{1+x})^n - (\sqrt{1-x})^n)^2 = \binom{n}{2}((\sqrt{1+x})^n - (\sqrt{1-x})^n)^2 \Rightarrow \text{koeficijent 3.člana je} \binom{n}{2}
\]

\[
\binom{n}{2} = 78
\]

\[
\frac{n(n-1)}{2 \cdot 1} = 78
\]

\[
n(n-1) = 156 = 13 \cdot 12 \Rightarrow n = 13
\]

Peti član:

\[
T_5 = \binom{13}{4}(\sqrt{1+x})^5(\sqrt{1-x})^4 = \binom{13}{4}(\sqrt{1+x})^5(\sqrt{1-x})^4 = 715(1+x)(1+x)^4(1-x)^2
\]

c) članove u razvoju binoma \((\sqrt[3]{3} + \sqrt{2})^5\) koji nisu iracionalni

rešenje:

\[
T_{k+1} = \binom{5}{k}(\sqrt[3]{3})^{5-k}(\sqrt{2})^k = \binom{5}{k}\frac{5-k}{3} \cdot 2^k
\]

Da bi članovi bili racionalni eksponenti:

\[
\frac{5-k}{3} \quad \text{I} \quad \frac{k}{2} \quad \text{moraju biti celi brojevi.}
\]

(k=0,1,2,3,4,5)

Zbog \(\frac{k}{2}-\text{ceo broj} \Rightarrow k\) je paran broj tj k=0,2,4.

Zamenom u \(\frac{5-k}{3}\) dobijamo da je to ceo broj samo za k=2.

Jedini racionalan član u razvoju binoma je 3.član: \(T_3 = \binom{5}{2}(\sqrt[3]{3})^3(\sqrt{2})^2 = 10 \cdot 3 \cdot 2 = 60\)

d) racionalne članove u razvoju binoma \((1 + \sqrt[4]{2})^{15}\)

rešenje:

\[
T_{k+1} = \binom{15}{k}(1)^{15-k}(\sqrt[4]{2})^k = \binom{15}{k}2^{\frac{k}{4}}
\]

Da bi članovi bili racionalni eksponent \(\frac{k}{4}\) mora biti ceo broj.

Zbog \(\frac{k}{4}-\text{ceo broj} \Rightarrow k\) je broj deljiv sa 4 => k=0, k=4, k=8 i k=12.
Racionalni članovi su: prvi, peti, deveti i trinaesti.

\[ T_1 = \binom{15}{0} \left( \frac{4}{2} \right)^0 = 1 \]

\[ T_5 = \binom{15}{4} \left( \frac{4}{2} \right)^4 = \binom{15}{4} \cdot 2 = 2730 \]

\[ T_9 = \binom{15}{8} \left( \frac{4}{2} \right)^8 = \binom{15}{7} \cdot 2^2 = 25740 \]

\[ T_{13} = \binom{15}{12} \left( \frac{4}{2} \right)^{12} = \binom{15}{3} \cdot 2^3 = 3640 \]

6. Odrediti \( x \) tako da:

a) 4. član u razvoju binoma \( \left( \sqrt[4]{x^{\log x + 1}} + \frac{1}{\sqrt{x}} \right)^6 \) bude jednak 200

rešenje:

\[ T_4 = \binom{6}{3} \left( \sqrt[4]{x^{\log x + 1}} \right)^3 \left( \frac{1}{\sqrt{x}} \right)^3 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \left( \sqrt[4]{x^{\log x + 1}} \right)^\frac{1}{2} \left( \frac{1}{x^{\frac{1}{12}}} \right)^\frac{3}{2} = 20x^{\frac{3}{2}\log(\log x + 1) + \frac{1}{4}} \cdot x^{\frac{3}{4}} = 20x^{\frac{3}{2}\log(\log x + 1) + \frac{1}{4}} \]

\[ 20x^{\frac{3}{2}\log(\log x + 1) + \frac{1}{4}} = 200 \]

\[ x^{\frac{3}{2}\log(\log x + 1) + \frac{1}{4}} = 10 \]

\[ \log x^{\frac{3}{2}\log(\log x + 1) + \frac{1}{4}} = \log 10 \]

\[ \left( \frac{3}{2\log x + 1} + \frac{1}{4} \right) \log x = 1 \]

Smena: \( \log x = t \)

\[ \left( \frac{3}{2(t + 1)} + \frac{1}{4} \right) t = 1 \]

\[ \frac{3t}{2(t + 1)} + \frac{t}{4} = 1 \]

\[ 6t + t^2 + t = 4t + 4 \]

\[ t^2 + 3t - 4 = 0 \]

\[ t_1 = -4 \]

\[ t_2 = 1 \]

(1) \( \log x = -4 \Rightarrow x = 10^{-4} = 0.0001 \)
b) 3. član u razvoju binoma \((x + x^{\log x})^5\) bude 1000000

rešenje:

\[
T_3 = \left(\frac{5}{2}\right)x^3\left(x^{\log x}\right)^2 = \frac{5 \cdot 4}{2 \cdot 1}x^3x^{2\log x} = 10x^3+2\log x
\]

\[10x^{3+2\log x} = 1000000 \div 10\]

\[x^{3+2\log x} = 100000 \div \log\]

\[\log x^{3+2\log x} = \log 10^5\]

\[(3 + 2\log x)\log x = 5\log 10\]

Smena: \(\log x = t\)

\[(3 + 2t)t = 5\]

\[2t^2 + 3t - 5 = 0\]

\[t_{1/2} = \frac{-3 \pm \sqrt{9 + 40}}{4} = \frac{-3 \pm 7}{4}\]

\[t_1 = \frac{-5}{2}\]

\[t_2 = 1\]

(1) \(\log x = \frac{-5}{2} \Rightarrow x = 10^{\frac{-5}{2}} = \sqrt{10^{-5}} = \frac{1}{\sqrt{10^5}} = \frac{1}{10^2\sqrt{10}} = \frac{\sqrt{10}}{10000}\)

(2) \(\log x = 1 \Rightarrow x = 10\)

c) 6. član u razvoju binoma \(\left(2^{\log_2 \sqrt{9^{x-1}+7}} + 2^{\frac{-1}{5}\log_2 \left(3^{x-1}+1\right)}\right)^7\) bude jednak 84.

rešenje:

\[
T_3 = \left(\frac{7}{5}\right)x^3\left(2^{\log_2 \sqrt{9^{x-1}+7}}\right)^2\left(2^{\frac{-1}{5}\log_2 \left(3^{x-1}+1\right)}\right)^5 = \left(\frac{7}{2}\right)2^{2\log_2 \sqrt{9^{x-1}+7}} \cdot 2^{\frac{-1}{5}\log_2 \left(3^{x-1}+1\right)} = 21 \cdot 2^{2\log_2 \sqrt{9^{x-1}+7} - \log_2 \left(3^{x-1}+1\right)}
\]

\[2^{2\log_2 \sqrt{9^{x-1}+7} - \log_2 \left(3^{x-1}+1\right)} = 84 \div 21\]

\[2^{2\log_2 \sqrt{9^{x-1}+7} - \log_2 \left(3^{x-1}+1\right)} = 4 \div \log_2\]

\[\log_2 2^{2\log_2 \sqrt{9^{x-1}+7} - \log_2 \left(3^{x-1}+1\right)} = \log_2 4\]

\[\left(2\log_2 \left(9^{x-1}+7\right)^{1/2} - \log_2 \left(3^{x-1}+1\right)\right)\log_2 2 = \log_2 4\]
\[
\log_2 \left( \frac{9^{x-1} + 7}{3^{x-1} + 1} \right) = \log_2 4
\]
\[
\log_2 \left( \frac{9^{x-1} + 7}{3^{x-1} + 1} \right) = \log_2 4
\]
\[
\frac{9^{x-1} + 7}{3^{x-1} + 1} = 4
\]
\[
9^{x-1} + 7 = 4(3^{x-1} + 1)
\]
\[
\frac{9^x}{9} + 7 = 4 \left( \frac{3^x}{3} + 1 \right)
\]
\[
\frac{3^{2x}}{9} + 7 = 4 \left( \frac{3^x}{3} + 1 \right)
\]
Smena: \(3^x = t > 0\)
\[
t^2 + 7 = \frac{4t}{3} + 4 \quad /: 9
\]
\[
t^2 + 63 = 12t + 36
\]
\[
t^2 - 12t + 27 = 0
\]
\[
t_1 = 9
\]
\[
t_2 = 3
\]
(1) \(3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2\)
(2) \(3^x = 3 \Rightarrow x = 1\)

d) 4. član u razvoju binoma \( \left( 10^{ \log \sqrt{x} } + 10^{\frac{-1}{\log x}} \right)^7 \) jednak 3500000

rešenje:
\[
T_4 = \binom{7}{3} \left( 10^{ \log \sqrt{x} } \right)^4 \left( 10^{\frac{-1}{\log x}} \right)^3 = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \cdot 10^{4 \log x^{1/2}} \cdot 10^{-3 \log x} = 35 \cdot 10^{2 \log x - \frac{3}{\log x}}
\]
\[
35 \cdot 10^{2 \log x - \frac{3}{\log x}} = 3500000 \quad /: 35
\]
\[
10^{2 \log x - \frac{3}{\log x}} = 100000 \quad /\log
\]
\[
\log 10 = \log 100000
\]
\[
\left( 2^{\log x} - \frac{3}{\log x} \right) \log 10 = \log 10^5
\]
2\log x - \frac{3}{\log x} = 5

Smena: \log x = t

2t - \frac{3}{t} = 5 \therefore t

2t^2 - 3 = 5t

2t^2 - 5t - 3 = 0

t_{1/2} = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4}

t_1 = 3 \quad \text{t}_2 = -\frac{1}{2}

(1) \log x = 3 \Rightarrow x = 10^3 = 1000

(2) \log x = -\frac{1}{2} \Rightarrow x = 10^{-\frac{1}{2}} = \frac{1}{\sqrt{10}}, \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}

7. Ako je u razvijenom obliku binoma (x + y)^n, drugi član 240, treći 720 a četvrti 1080 odrediti x, y i n.

rešenje:

T_2 = \binom{n}{1}x^{n-1}y = nx^{n-1}y

nx^{n-1}y = 240

T_3 = \binom{n}{2}x^{n-2}y^2 = \frac{n(n-1)}{2}x^{n-2}y^2

\frac{n(n-1)}{2}x^{n-2}y^2 = 720 \Rightarrow n(n-1)x^{n-2}y^2 = 1440

T_4 = \binom{n}{3}x^{n-3}y^3 = \frac{n(n-1)(n-2)}{6}x^{n-3}y^3

\frac{n(n-1)(n-2)}{6}x^{n-3}y^3 = 1080 \Rightarrow n(n-1)(n-2)x^{n-3}y^3 = 6480

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nx^{n-1}y = 240 \quad (1)

n(n-1)x^{n-2}y^2 = 1440 \quad (2)

n(n-1)(n-2)x^{n-3}y^3 = 6480 \quad (3)

\frac{(2)}{(1)} \Rightarrow \frac{n(n-1)x^{n-2}y^2}{nx^{n-1}y} = \frac{1440}{240} \Rightarrow (n - 1)\frac{y}{x} = 6 \quad (4)

\frac{(3)}{(2)} \Rightarrow \frac{n(n-1)(n-2)x^{n-3}y^3}{n(n-1)x^{n-2}y^2} = \frac{6480}{1440} \Rightarrow (n - 2)\frac{y}{x} = \frac{9}{2} \quad (5)

Iz (4) i (5):
\[(n - 2) \frac{y}{x} = \frac{9}{2}\]
\[(n - 1) \frac{y}{x} - \frac{y}{x} = \frac{9}{2}\]
\[6 - \frac{y}{x} = \frac{9}{2}\]
\[y = 3\]
\[x = \frac{2}{2}\]

Zamenom u (4)
\[(n - 1) \frac{3}{2} = \frac{9}{2} \Rightarrow n - 2 = 3 \Rightarrow n = 5\]

5\(x^4y = 240\)
\(x^4y = 48\)
\(x^4 \cdot \frac{3}{2}x = 48\)
\(x^5 = 48 \cdot \frac{2}{3}\)
\(x^5 = 32 = 2^5\)
\[x = 2\]
\[y = \frac{3}{2}\]
\[y = \frac{3}{2}\]
\[y = 3\]

8. Dokazati da je zбир svih binomnih koeficijenata u razvoju binoma jednak \(2^n\).

Rešenje:
Zbir binomnih koeficijenata je:
\[
\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}
\]
Ovaj zbir se dobija razvojem binoma \((1 + 1)^n\).
Odnosno
\[
\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = (1 + 1)^n = 2^n
\]